

Stability conditions for fermionic Ising spin-glass models in the presence of a transverse field

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Abstract

The stability of spin-glass (SG) phase is analyzed in detail for a fermionic Ising SG (FISG) model in the presence of a magnetic transverse field Γ . The fermionic path integral formalism, replica method and static approach have been used to obtain the thermodynamic potential within one step replica symmetry breaking ansatz. The replica symmetry (RS) results show that the SG phase is always unstable against the replicon. Moreover, the two other eigenvalues λ_{\pm} of the Hessian matrix (related to the diagonal elements of the replica matrix) can indicate an additional instability to the SG phase, which enhances when Γ is increased. Therefore, this result suggests that the study of the replicon can not be enough to guarantee the RS stability in the present quantum FISG model, especially near the quantum critical point. In particular, the FISG model allows changing the occupation number of sites, so one can get a first order transition when the chemical potential exceeds a certain value. In this region, the replicon and the λ_{\pm} indicate instability problems for the SG solution close to all range of first order boundary.

1 Introduction

The interplay between disorder, frustration and quantum effects presents several challenging issues. For instance, the infinite-range Ising spin-glass model in a transverse magnetic field has been studied with various techniques that show controversial results. It is well known that the classical Sherrington-Kirkpatrick (SK) model [1] presents a continuous phase transition in which the spin-glass (SG) phase has the free energy landscape composed by many almost degenerated thermodynamic states separated by infinitely high barriers [2]. The controversy is whether or not the quantum tunneling between free energy barriers activated by the transverse field is able to restore the replica symmetry (RS) in the infinite-range Ising spin-glass model. [3, 4, 5, 6, 7, 8]. To answer this question, the

usual procedure is to investigate the behavior of the so-called replicon (the transversal eigenvalue of the Hessian matrix) to check the local stability of RS solution. No much attention is given to the longitudinal eigenvalues since, as in the classical case, they would not give any additional important information in such an analysis [3, 6]. Nevertheless, there is, at least, one formulation of the infinite-range Ising spin-glass problem, the fermionic Ising spin-glass (FISG) model [9, 10, 11], in which the role of the quantum effects on the longitudinal eigenvalues could also be important.

The FISG model is defined by spin operators which are represented by bilinear combinations of fermion operators of creation and destruction. That makes this model a quite adequate framework to study the competition between spin-glass and, for instance, Kondo effect or superconductivity. [12, 13, 14]. Those spin operators act on Fock space with four states per site, two of them non-magnetic. In fact, this model can also be formulated in two versions [15, 16]. In the first one ($2S$ model), it is used a restriction in the number operator that eliminates the contribution of nonmagnetic states. That would be equivalent to study the problem in the spin space. In the second version ($4S$ model), magnetic and nonmagnetic states are admitted. In this case, an important aspect is the connection between spin and charge correlations [17]. That can be seen clearly, for instance, in the relationship between n (the average of occupation of fermions per site) and the diagonal replica matrix elements [13, 17]. Therefore, in the $4S$ model, variations of charge occupation can influence the magnetic properties not only leading the onset of the spin-glass phase, but also changing the nature of the phase boundary. For instance, the phase diagram of the $4S$ model presents a tricritical point [17] and a reentrance in the first order boundary phase for specific values of the chemical potential μ [14]. The mentioned connection can also appear in the behavior of longitudinal eigenvalues of the Hessian matrix. Indeed, in the $4S$ model these eigenvalues present a non-trivial behavior, in which they become complex [13, 17].

The presence of a transverse field Γ produces important changes in the phase diagram of $2S$ and $4S$ models. For instance, the increase of Γ leads the freezing temperature (T_f) of both models to a Quantum Critical Point (QCP) at $\Gamma = \Gamma_c$ [8, 15]. Particularly, for the $4S$ -model, when $\mu \neq 0$, the increase of Γ not only depresses the second order part of T_f and the position of the tricritical point but also destroys the reentrance in the first order part of T_f [14]. In other words, Γ induces quantum spin flipping mechanisms that affect the replica matrix elements strongly, in particular, the diagonal ones. Because of that, Γ could interfere in the replicon (λ_{AT}) and, it should be remarked, in the longitudinal eigenvalues (λ_{\pm}) of Hessian matrix for both $2S$ and $4S$ models.

The previous discussion leads to the question: how exactly would Γ affect λ_{\pm} in these two models? In the $2S$ model, the diagonal replica matrix elements become dependent on temperature when $\Gamma > 0$, which could reflect on the stability of the SG phase. Thus, it can be a mechanism to change the behavior of λ_{\pm} since, at $\Gamma = 0$, this model reproduces the basic features of the classical SK one. One can also remark that Γ in the $2S$ model produces effects which are limited to the magnetic properties. Nevertheless, that is not the case for the

4S one. In this model, there is a mixing between spin and charge correlation functions which is important to determine not only magnetic properties [14] but also charge ones [13]. In this particular model, even for $\Gamma = 0$, the diagonal replica matrix elements have a relevant role for the SG solution when μ is increased, which can lead to a non-trivial behavior of λ_{\pm} , as discussed previously. Consequently, one can expect that the combined effects of variations of Γ and μ could enhance the non-trivial behavior of λ_{\pm} . Actually, the variations of these independent parameters create a complicated interplay between charge and spin degrees of freedom in which both magnetic properties and charge distribution, for instance, are affected. However, such redistribution of charge can influence the magnetic correlation functions themselves. As a result, the behavior of λ_{\pm} in the 4S model can be considerably more complicated than the 2S one.

Therefore, the main purpose of this paper is to study in detail the RS stability for both the 2S and 4S models in the presence of a transverse magnetic field Γ . In particular, for the 4S model, special attention is given to investigate effects of combined variations from Γ and μ on λ_{AT} and λ_{\pm} . Furthermore, it is also obtained the behavior of n inside the SG phase to clarify how spin and charge correlation functions influence each other. In order to accomplish these goals, the partition function is obtained in the path integral formalism in which the spin operators are given as bilinear combinations of Grassmann fields. The thermodynamic potential is found within the static approach (SA) and the one step replica symmetry breaking (1S-RSB) scheme, which allows to derive n . In particular, to obtain the 2S model, it is introduced the same restraint used in Refs. [10, 15] to avoid the two nonmagnetic eigenstates. Finally, the local stability of the RS solution is studied by obtaining λ_{AT} and λ_{\pm} .

The present work the static approach which neglects time fluctuations of spin-spin correlation functions is used [18]. It is clear that the SA is unable to provide reliable quantitative results for the thermodynamics at very low temperature [19]. However, it has also been shown by Grempel and Rozemberg [20] that static ansatz gives reliable results when the temperature is not too low. In fact, the results found in Ref. [20] for the spin-spin correlation function $Q(\tau)$ indicate a behavior which is approximate to the same of their classical counterpart within an interval of temperature around a continuous SG transition. On the other hand, when a first order transition is present, it would be expected that SA could produce naturally reasonable results around the transition [19]. This interval of reliability for both kind of transition justifies the use of SA for purposes of this work.

It is important to mention that the scenario described for the 4S-model when $\Gamma = 0$ has been already found in another model. The classical Ghatak-Sherrington (GS) model [21] is a particularly interesting example which displays a spin-glass first order phase transition [22, 23, 24, 25]. In the GS model, there are three distinct eigenvalues of the Hessian matrix in the limit of number of replicas $n \rightarrow 0$. As in the classical SK model [26], the replicon (λ_{AT}) is negative in the whole SG region. However, other two eigenvalues (λ_{\pm}) can be complex or negative [23, 24, 25]. Actually, there is a close relationship between GS and FISG models [27]. In particular, the thermodynamics described in the GS

model can also be found in the $4S$ -model, such as the reentrance in the first order boundary phase. This similarity of the thermodynamics between these two models is obtained by a mapping of their spin-glass order parameters equations, within the SA and RS solution, given by a relationship between the chemical potential μ and the anisotropic constant D of the GS model [13]. In that sense, the GS model can be a reference to check the correctness of results obtained in the $4S$ -model when $\Gamma \rightarrow 0$.

The organization of this article is as follows: in section 2, the $FISG$ model in a Γ field is presented. The thermodynamic potential is obtained within the SA ansatz and $1S$ - RSB solution. In this section, it is also done a detailed study of the local stability of the RS solution for the present $FISG$ model. Section 3 is devoted to describe the numerical results. Section 4 is left for conclusions.

2 General Formulation

The infinite-range Ising spin-glass model in the presence of a transverse magnetic field is described by:

$$\hat{H} = - \sum_{i \neq j} J_{ij} \hat{S}_i^z \hat{S}_j^z - 2\Gamma \sum_i \hat{S}_i^x \quad (1)$$

where the sums are run over the N sites of a lattice. The exchange interaction J_{ij} among all pairs of spins is assumed to be a random variable with a Gaussian distribution

$$P(J_{ij}) = \sqrt{N/(32\pi J^2)} \exp(-J_{ij}^2 N/32J^2). \quad (2)$$

The spin operators are defined as [15]:

$$\hat{S}_i^z = \frac{1}{2}[\hat{n}_{i\uparrow} - \hat{n}_{i\downarrow}] , \quad \hat{S}_i^x = \frac{1}{2}[c_{i\uparrow}^\dagger c_{i\downarrow} + c_{i\downarrow}^\dagger c_{i\uparrow}] \quad (3)$$

where $\hat{n}_{i\sigma} = c_{i\sigma}^\dagger c_{i\sigma}$ is the number operator, $c_{i\sigma}^\dagger$ ($c_{i\sigma}$) is fermion creation (destruction) operator and $\sigma = \uparrow$ or \downarrow indicate the spin projections.

The Hamiltonian given in Eq. (1) is defined on the Fock space where there are four states per site: one state with no fermion, two states with a single fermion and one state with two fermions. Consequently, there are two nonmagnetic states. This work considers two models: the $4S$ model that allows the four possible states per site and the $2S$ model, which restricts the spin operators to act on a space where the nonmagnetic states are forbidden. Therefore, the $2S$ model requires a restriction to remove the contribution of these nonmagnetic states. It can be obtained by computing only sites occupied by one fermion ($n_{i\uparrow} + n_{i\downarrow} = 1$ at every site) in the partition function trace [10, 15].

In this fermionic problem the partition function is expressed by use of the Lagrangian path integral formalism in terms of anticommuting Grassmann fields (ϕ and ϕ^*) [15]. The restriction in the $2S$ -model is obtained by using the Kronecker delta function [10, 15]. Therefore, adopting an integral representation

for this delta function, one can express the partition function for both models in a compact form:

$$Z\{y\} = e^{\frac{s-2}{2}N\beta\mu} \int D(\phi^*\phi) \prod_j \frac{1}{2\pi} \int_0^{2\pi} dx_j e^{-y_j} e^{A\{y\}} \quad (4)$$

where

$$A\{y\} = \int_0^\beta d\tau \left\{ \sum_{j,\sigma} \phi_{j\sigma}^*(\tau) \left[\frac{\partial}{\partial \tau} + \frac{y_j}{\beta} \right] \phi_{j\sigma}(\tau) - H(\phi_{j\sigma}^*(\tau), \phi_{j\sigma}(\tau)) \right\}, \quad (5)$$

$\beta = 1/T$ (T is the temperature), $y_j = ix_j$ for the $2S$ -model or $y_j = \beta\mu$ for the $4S$ -model, μ is the chemical potential and s is the state number per site allowed in each model.

Now, it follows the standard procedures in which the replica method [2],

$$\beta\Omega = -\frac{1}{N} \langle \ln Z\{y\} \rangle_{J_{ij}} = -\frac{1}{N} \lim_{n \rightarrow 0} \frac{\langle Z\{y\}^n \rangle_{J_{ij}} - 1}{n} \quad (6)$$

is used to get the configurational averaged thermodynamic potential. The replicated partition function $\langle Z\{y\}^n \rangle_{J_{ij}}$ is then given by:

$$\langle Z\{y\}^n \rangle_{J_{ij}} = e^{\frac{s-2}{2}N\beta\mu} \mathcal{N} \int_{-\infty}^{\infty} \prod_{\alpha,\gamma}^n dq_{\alpha\gamma} \int_{-\infty}^{\infty} \prod_{\alpha}^n dp_{\alpha} \exp[N\Omega_n(q_{\alpha\gamma}, p_{\alpha})] \quad (7)$$

where $\mathcal{N} = (\beta J \sqrt{N/2\pi})^{n(n+1)/2}$ and $\alpha = 1, 2, \dots, n$ is the replica index. Within the static approximation, one has:

$$\Omega_n(q_{\alpha\gamma}, p_{\alpha}) = -\beta^2 J^2 \sum_{(\alpha,\gamma)} q_{\alpha\gamma}^2 - \frac{\beta^2 J^2}{2} \sum_{\alpha} p_{\alpha}^2 + \ln \Lambda\{y\} \quad (8)$$

where the Fourier representation is used to express:

$$\Lambda\{y\} = \prod_{\alpha} \frac{1}{2\pi} \int_0^{2\pi} dx_{\alpha} e^{-y_{\alpha}} \int D[\phi_{\alpha}^*, \phi_{\alpha}] \exp[H_{eff}], \quad (9)$$

$$H_{eff} = \sum_{\alpha} A_{0\Gamma}^{\alpha} + 4\beta^2 J^2 \left(\sum_{\alpha} p_{\alpha} S_{\alpha}^z S_{\alpha}^z + 2 \sum_{(\alpha,\gamma)} q_{\alpha\gamma} S_{\alpha}^z S_{\gamma}^z \right) \quad (10)$$

with the convention that (α, γ) indicates a distinct pair of replicas and the definitions:

$$A_{0\Gamma}^{\alpha} = \sum_{\omega} \underline{\varphi}_{\alpha}^{\dagger}(\omega) (i\omega + y_{\alpha} + \beta \underline{\sigma}^x) \underline{\varphi}_{\alpha}(\omega), \quad S_{\alpha}^z = \frac{1}{2} \sum_{\omega} \underline{\varphi}_{\alpha}(\omega) \underline{\sigma}^z \underline{\varphi}_{\alpha}(\omega), \quad (11)$$

$\underline{\varphi}_{\alpha}^{\dagger}(\omega) = (\phi_{\uparrow\alpha}^*(\omega) \quad \phi_{\downarrow\alpha}^*(\omega))$, $\omega = \pm\pi, \pm 3\pi, \dots$. $\underline{\sigma}^x$ and $\underline{\sigma}^z$ are the Pauli matrices.

Therefore, in the 1S-RSB Parisi scheme [28], the thermodynamic potential is written from Eq. (6) as

$$\begin{aligned} \beta\Omega = & \frac{(\beta J)^2}{2}[(m-1)q_1^2 - mq_0^2 + p^2] - \frac{(s-2)}{2}\beta\mu \\ & - \frac{1}{m} \int Dz \ln \left\{ \int Dv [K(z, v)]^m \right\} - \ln 2 \end{aligned} \quad (12)$$

where

$$K(z, v) = \frac{(s-2)}{2} \cosh(\beta\mu) + \int D\xi \cosh[\sqrt{\Delta(z, v, \xi)}], \quad (13)$$

with $\Delta(z, v, \xi) = [\beta h(z, v, \xi)]^2 + (\beta\Gamma)^2$ and

$$h(z, v, \xi) = J\sqrt{2}(\sqrt{q_0}z + \sqrt{q_1 - q_0}v + \sqrt{p - q_1}\xi), \quad (14)$$

where $Dx = dx e^{-x^2/2}/\sqrt{2\pi}$ ($x = z, v$ or ξ). In Eqs. (12) and (14), q_0 and q_1 are the 1S-RSB order parameters and $p = q_{\alpha\alpha} = \langle S_\alpha^z S_\alpha^z \rangle$ is the diagonal replica spin-spin correlation that is related to the diagonal replica matrix elements. The parameters q_0 , q_1 , p and m are given by the extreme condition of the thermodynamic potential Eq. (12). The RS solution is recovered when $q_0 = q_1 (\equiv q)$ and $m = 0$.

Particularly, in the 4S model, the chemical potential μ controls the average occupation of fermions per site $n = \langle n_\uparrow + n_\downarrow \rangle$ which is obtained as:

$$n = 1 + \tanh(\beta\mu) \left\{ (1-p) - (\beta\Gamma)^2 \int Dz \frac{\int Dv [K(z, v)]^{m-1} \int D\xi [\phi(z, v, \xi)]}{\int Dv [K(z, v)]^m} \right\} \quad (15)$$

with

$$\phi(z, v, \xi) = \frac{\cosh \sqrt{\Delta(z, v, \xi)}}{\Delta(z, v, \xi)} - \frac{\sinh \sqrt{\Delta(z, v, \xi)}}{\Delta^{3/2}(z, v, \xi)}. \quad (16)$$

The local stability of the RS solution for the FISG model is studied following close to de Almeida-Thouless analysis [24]. For this purpose, the stationary points (Eq. (8)) are arbitrarily perturbed by independent quantities $(\xi_\alpha, \eta_{\alpha\gamma})$. The deviation $\Delta_n(\xi_\alpha, \eta_{\alpha\gamma}) = \Omega_n(p_\alpha, q_{\alpha\gamma}) - \Omega_n(p_\alpha + 2\xi_\alpha, q_{\alpha\gamma} + \eta_{\alpha\gamma})$ can be obtained expanding Δ_n up to the second order in $(\xi_\alpha, \eta_{\alpha\gamma})$:

$$\Delta_n = \sum_{\alpha, \gamma} G_{\alpha, \gamma} \xi_\alpha \xi_\gamma + 2 \sum_{\alpha, (\gamma\nu)} G_{\alpha, \gamma\nu} \xi_\alpha \eta_{\gamma\nu} + \sum_{(\alpha\beta), (\gamma\nu)} G_{\alpha\beta, \gamma\nu} \eta_{\alpha\beta} \eta_{\gamma\nu} \quad (17)$$

where

$$G_{\alpha, \gamma} = \frac{\delta_{\alpha, \gamma}}{\beta^2 J^2} - 16 \langle S_\alpha^z S_\alpha^z S_\gamma^z S_\gamma^z \rangle + 16 \langle S_\alpha^z S_\alpha^z \rangle \langle S_\gamma^z S_\gamma^z \rangle = (A - B) \delta_{\alpha, \gamma} + B \quad (18)$$

$$G_{\alpha, \gamma\nu} = -16 \langle S_\alpha^z S_\alpha^z S_\gamma^z S_\nu^z \rangle + 16 \langle S_\alpha^z S_\alpha^z \rangle \langle S_\gamma^z S_\nu^z \rangle = \begin{cases} C & \text{if } \alpha = \gamma \text{ or } \nu \\ D & \text{if } \alpha \neq \gamma \text{ and } \nu \end{cases} \quad (19)$$

$$G_{\alpha\beta,\gamma\nu} = \frac{\delta_{\alpha\beta,\gamma\nu}}{2\beta^2 J^2} - 16\langle S_\alpha^z S_\beta^z S_\gamma^z S_\nu^z \rangle + 16\langle S_\alpha^z S_\beta^z \rangle \langle S_\gamma^z S_\nu^z \rangle = \begin{cases} P & \text{if } \alpha\beta = \gamma\nu \\ Q & \text{if } \alpha = \gamma(\beta \neq \nu) \\ R. & \end{cases} \quad (20)$$

The eigenvalues associated with the Hessian matrix (17) are known [26]. In the limit $n \rightarrow 0$, there are three distinct eigenvalues:

$$\lambda_\pm = [A - B + (P - 4Q + 3R) \pm \sqrt{U}]/2 \quad (21)$$

and

$$\lambda_{AT} = P - 2Q + R \quad (22)$$

where

$$U = [(A - B) - (P - 4Q + 3R)]^2 - 8(C - D)^2, \quad (23)$$

$$P - 4Q + 3R = \frac{1}{2\beta^2 J^2} - \int Dz [\varphi_2^2(z) - 4\varphi_2(z)\varphi_1^2(z) + 3\varphi_1^4(z)], \quad (24)$$

$$P - 2Q + R = \frac{1}{2\beta^2 J^2} - \int Dz [\varphi_2(z) - \varphi_1^2(z)]^2, \quad (25)$$

$$A - B = \frac{1}{\beta^2 J^2} - \int Dz [\varphi_4(z) - \varphi_2^2(z)], \quad (26)$$

$$C - D = \int Dz [\varphi_2(z)\varphi_1^2(z) - \varphi_3(z)\varphi_1(z)], \quad (27)$$

with

$$\varphi_n(z) = \left[\int D\xi \frac{\partial^n}{\partial h^n} \left(\cosh \sqrt{\bar{\Delta}(z, \xi)} \right) \right] / \bar{K}(z), \quad (28)$$

$\bar{\Delta}(z, \xi) = \bar{h}^2(z, \xi) + \beta^2 \Gamma^2$ with $\bar{h}(z, \xi) = \beta J \sqrt{2}(\sqrt{q}z + \sqrt{p-q}\xi)$, and

$$\bar{K}(z) = \frac{s-2}{2} \cosh \beta\mu + \int D\xi \cosh \sqrt{\bar{\Delta}(z, \xi)}. \quad (29)$$

In that case, the stable RS solution would occur when the Hessian eigenvalues λ_\pm and λ_{AT} are all positive. As one can see, different from the classical SK model, in the FISG model, the replica diagonal elements have an important role in the stability of the RS solution. Particularly, the condition for all eigenvalues of the Hessian matrix to be nonnegative in the paramagnetic solution ($q = 0$) is used to locate the tricritical temperature $T_{tc}(\Gamma)$ at (for detail, see Ref. [14]):

$$T_{tc}/J = \frac{\sqrt{2}}{3} \varphi_4 / \varphi_2 \quad (30)$$

where φ_2 and φ_4 are defined in Eq. (28) when $q = 0$ and $p = T_{tc}/J\sqrt{2}$.

3 Results

The numerical solutions of the saddle point order parameters equations allow one to build phase diagrams (T/J) (T is the temperature) and Γ/J (see, for instance, Refs. [15, 16]). In the 4S model, it is also taken into account μ/J to build phase diagrams (see, for instance, Ref. [14]). The parameter J is related to the variance of the Gaussian random coupling J_{ij} given in Eq. (2). Particularly, for the 4S case, the first order boundary has been located using the procedure introduced in Ref. [25] for the GS model. The Hessian eigenvalues (Eqs. (21-22)) are inserted inside these phase diagrams.

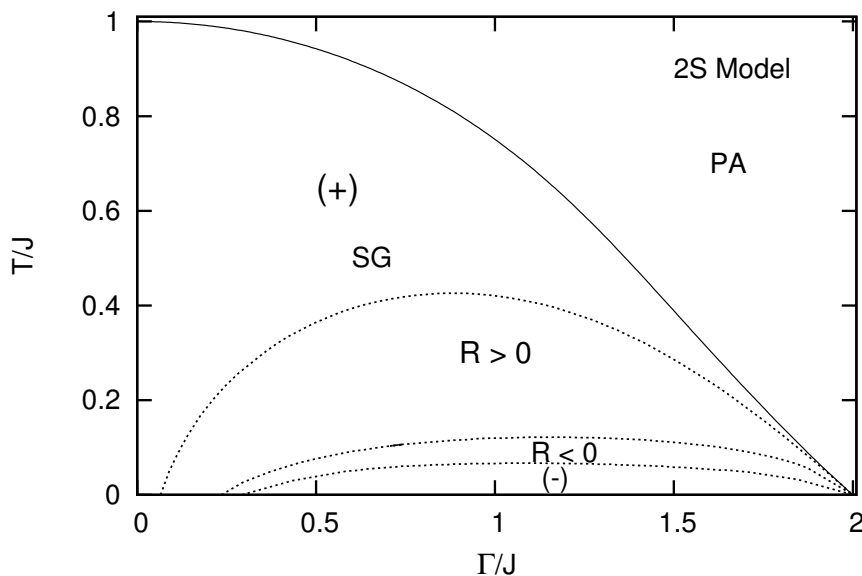


Figure 1: Phase diagram T/J versus Γ/J for the 2S model showing the behavior of the eigenvalues λ_+ and λ_- . The notation (+) ((-)) indicates a region where both eigenvalues λ_+ and λ_- are real positive (negative). In the region indicated by $R > 0$ ($R < 0$), these eigenvalues become complex with their real part positive (negative). The solid line indicates a second order transition.

Fig. 1 shows a phase diagram T/J versus Γ/J for the 2S model. This is equivalent to study the problem in the spin space, as discussed previously. The SG phase boundary is followed by Almeida-Thouless line (defined as $\lambda_{AT} = 0$) which means that λ_{AT} is negative in the whole SG phase [15]. For very small Γ/J , eigenvalues λ_+ and λ_- (denoted by λ_{\pm}) are real positive (indicated by (+) in Fig. 1). When Γ is enhanced, the contribution from the replica diagonal matrix elements for the SG solution starts to increase. As a consequence, eigenvalues λ_{\pm} become complex conjugated pairs. In regions indicated by R (see Fig. 1), λ_{\pm} assume complex values with their real part positive ($R > 0$)

or negative ($R < 0$). At very low temperature, λ_{\pm} are real negative (indicate by $(-)$ in Fig. 1). The onset of regions with $R > 0$ and $R < 0$ always occurs below $\lambda_{AT} = 0$. However, when Γ increases, boundaries of such regions, as well as that one with $(-)$, become increasingly close to the AT line. In fact, these boundaries go toward the QCP given by $\Gamma_c = 2J$, which suggest that the positivity of λ_{AT} in the FISG model could not be enough to guarantee the replica symmetry stability near the QCP .

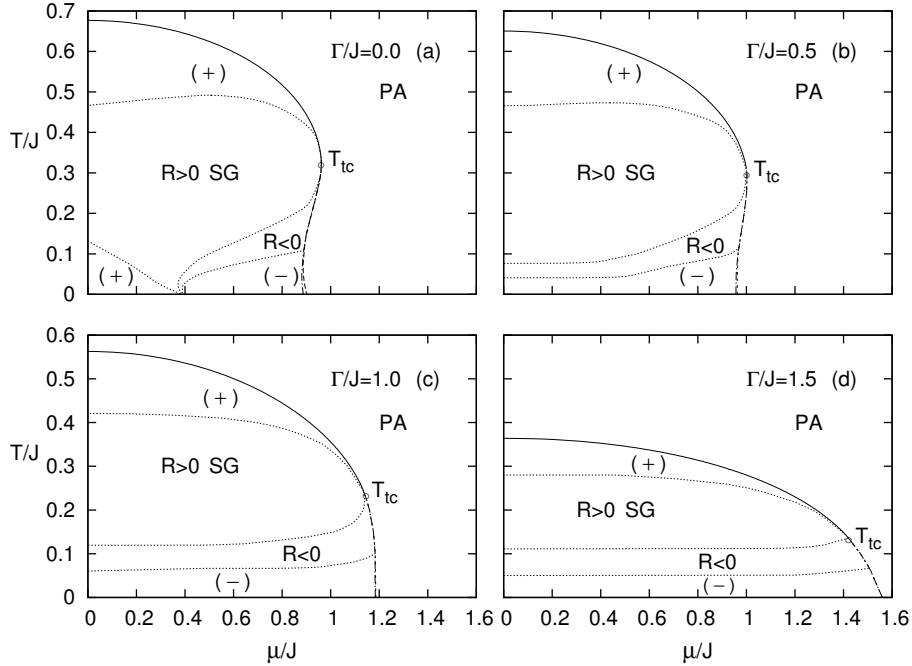


Figure 2: Phase diagrams T/J versus Γ/J showing the behavior of eigenvalues λ_{+} and λ_{-} for the 4S model and several values of μ/J . It is used the same convention as Fig. 1 for the instability regions of the RS solution. The solid lines indicate second order phase transition. Dashed lines indicate a first order phase transition, in which results within RS and 1S-RSB solution are compared. T_{tc} indicates the tricritical point.

In the 4S model, the combined effects of μ and Γ on λ_{AT} and the λ_{\pm} can be seen in Fig. 2. This figure shows phase diagrams T/J versus Γ/J for several values of μ/J . For all values chosen for μ/J , the freezing temperature T_f decreases toward a QCP when $\Gamma \rightarrow \Gamma_c$, as in the 2S-model. The half-filling situation ($\mu = 0$) is exhibited in Fig. 2(a). In this case, the SG solution is always unstable against the replicon stability analysis. However, in contrast with the 2S-model, it now appear a region with $R > 0$ even for $\Gamma = 0$. Fig. 2(b) presents the same characteristics found in Fig. 2(a), except there is no region with $(+)$

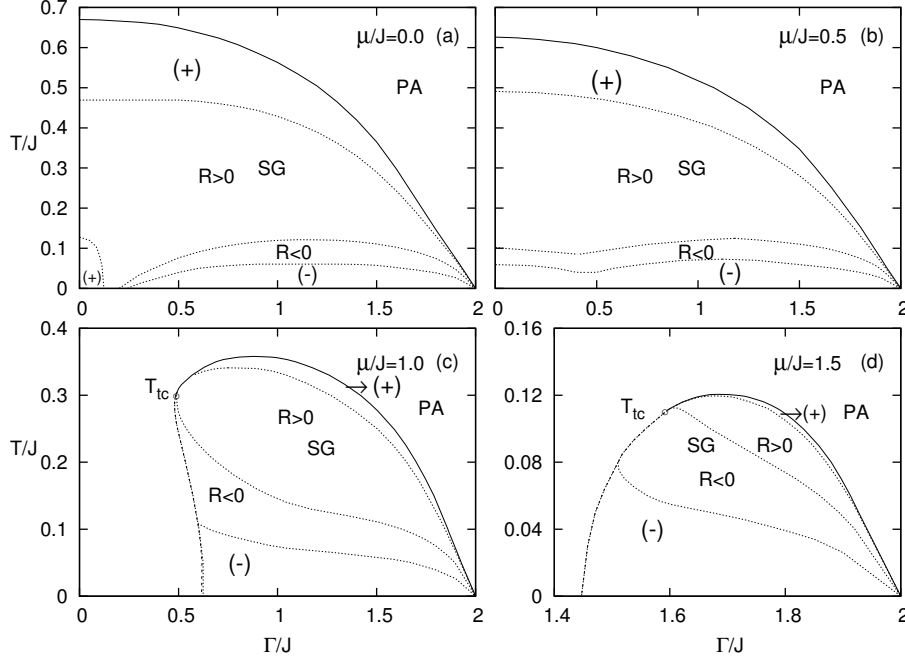


Figure 3: Phase diagrams T/J versus μ/J for the 4S model with for several values of Γ/J . The same convention as Fig. 1 is used for the instability regions of the RS solution. The solid lines indicate second order transition while dashed lines indicate a first order transition, in which the RS and 1S-RSB solutions are compared.

at low Γ/J and T/J . Nevertheless, for higher values of μ/J , as shown in Figs. 2(c)-(d), the consequences for SG instability from the interplay between charge and spin correlations become more important. For instance, when $\mu/J = 1$ (see Fig. 2(c)), there are no SG solutions for $0 \leq \Gamma \lesssim 0.5/J$. When $\Gamma \gtrsim 0.5J$, with the arising of these solutions, there is also a change in the nature of the SG boundary phase with the onset of a tricritical point at temperature T_{tc} . Below T_{tc} , a first order boundary is located in RS and 1S-RSB schemes, but, without any important difference between them [14]. Regarding the behavior of λ_{AT} at high values of μ/J , it is found that the replicon remains negative in the entire SG phase, as previously observed in Figs. 2(a)-(b). However, there is an enlargement of regions with $R > 0$, $R < 0$ and $(-)$. Particularly, close to the first order boundary phase, there are only regions with $R < 0$ or $(-)$. On the contrary, close to the second order boundary phase, the region with $(+)$ becomes smaller. Finally, in Fig. 2(d), the SG phase boundary is strongly decreased as compared with Fig. 2(c). The important point in this last case is that the regions with $R < 0$ or $(-)$ occupy almost the whole SG phase.

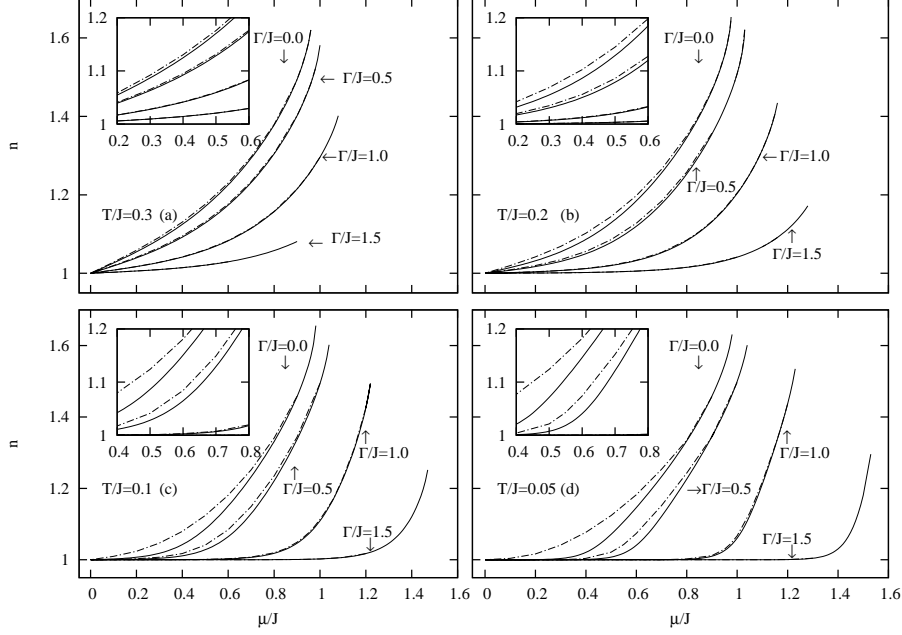


Figure 4: Average occupation n versus μ/J for several values of T/J and Γ/J of the SG solution. The dashed-dotted lines indicate the 1S-RSB solution. The insets show in detail the difference between the value of n in RS and 1S-RSB solution.

The combined effects of Γ and μ can also be seen in a different perspective. In Fig. 3, the phase diagrams are built now in T/J - μ/J surfaces for several values of Γ/J . Fig. 3(a) shows a phase diagram for $\Gamma = 0$ with regions $R > 0$, $R < 0$ and $(-)$. This result can be compared with Ref. [25] using the mapping introduced in Ref. [13]. The phase boundary transition at T_f is continuous at low μ/J . When μ/J increases, a tricritical point appears at T_{tc} and, at lower temperature, the first order part of T_f presents a reentrance (see discussion in Ref. [14]). Close to first order boundary, besides the usual replicon instability, the eigenvalues λ_{\pm} become complex with $R < 0$ as shown in Fig. 2. The enhancement of Γ , shown in Figs. 3(b)-(d), decreases the second order boundary phase and the tricritical point as well as it suppresses the reentrance in the first order one [14]. The behavior of λ_{AT} remains basically the same found in Fig. 2. The SG phase is always replica symmetry breaking. However, again, the region with $(+)$ decreases, while those ones with $R > 0$, $R < 0$ and $(-)$ increase. It should be emphasized that, close to the first order transition, it is observed, again, a region where, at same time, the replicon and the λ_{\pm} violate the replica stability condition.

The complicated connection between charge and spin correlations due to the presence of Γ can also be seen in the average occupation of fermions per site n

(see Eqs. (15)-(16)). Figure 4 shows the behavior of n within RS and $1S$ - RSB schemes as a function of μ/J for several isotherms and Γ/J . For $\Gamma = 0$, when temperature is decreased, n remains at the half-filling even for $\mu/J \neq 0$. This is consistent with an earlier result [16, 17], which suggests that the occupation of nonmagnetic states is exponentially small when T is decreased. However, it is also true that this effect is weakened when $1S$ - RSB solution is used, particularly, at lower T [13]. Nevertheless, the presence of Γ enforces the tendency to preserve the half-filling occupation, as can be seen, for higher T in Fig. 4(a). Indeed, this effect is enhanced when temperature is lowered as shown in Figs. 4(b)-(d). Most important, when Γ is turned on, the differences between RS and $1S$ - RSB solutions become irrelevant. In other words, Γ enforces the effect of temperature preserving n at the half-filling for both RS and $1S$ - RSB levels of description.

4 Conclusions

In this work, it has been studied the stability of spin-glass phase in the FISG model when a magnetic transverse field Γ is applied and, particularly, λ_{\pm} (the longitudinal eigenvalues of the Hessian matrix). The reason is that the presence of Γ produces important effects in the diagonal replica matrix elements which are closely related to the behavior of λ_{\pm} . In fact, the FISG model can be presented in two formulations, the so-called $4S$ and $2S$ models [9, 16]. In the $4S$ model, the original four states of the Fock space are maintained. In the $2S$ one, the nonmagnetic states are not allowed. Particularly, this means that in the $4S$ model there is a connection between charge and spin degrees of freedom [13, 29].

The results presented in Figs. 1-2 display two distinct situations for λ_{AT} and λ_{\pm} . For the $2S$ and $4S$ models, the results show that the replicon λ_{AT} is always real negative in the entire SG phase for any value of Γ/J . On the other hand, for the $2S$ model (see Fig. 1), the enhancement of Γ/J produces regions where λ_{\pm} becomes complex with real part positive ($R > 0$), negative ($R < 0$) or even real negative ($(-)$). Furthermore, all boundary lines of such regions converge to the QCP when $\Gamma \rightarrow \Gamma_c$. In the $4S$ model (see Fig. 2), λ_{\pm} also present regions with $R > 0$, $R < 0$ and $(-)$. In fact, there are other similarities between the two models. When $\Gamma \rightarrow \Gamma_c$, the boundary lines of these regions have the same behavior already found in the $2S$ model. Most important, this result is independent of a particular value of μ/J . Nevertheless, for $\Gamma < \Gamma_c$, when $0 \leq \mu < J$, the regions with $R < 0$ and $(-)$ are relatively small and they appear only at lower temperature. By contrast, the situation is different when $\mu \gtrsim J$. Particularly, regions with $R < 0$ and $(-)$ are considerably enlarged while that one with $(+)$ is decreased. It should be remarked that close to the first order boundary phase, besides the region with $(-)$, there is only region with $R < 0$.

The role of Γ described in the previous paragraph for the $4S$ model is confirmed in Fig. 3. There, the increase of μ produces regions with $R > 0$, $R < 0$ and $(-)$ as expected, for instance, from the mapping with the GS model when

$\Gamma = 0$ [13]. However, the increase of Γ tends to depress T_f , the tricritical point T_{tc} and to destroy the reentrance in the first boundary phase [14]. Such increase also depresses boundary lines of regions with $R > 0$, $R < 0$ and $(-)$. The important point is that these boundary lines follow the change in location of the T_{tc} . Therefore, no matter the value of μ , the increase of Γ tends to reproduce the scenario in which all boundary lines of regions $R > 0$, $R < 0$ and $(-)$ are decreasing towards zero temperature.

The behavior of n exhibited in Fig. 4 can clarify the role of Γ in the problem as source of similarities between the $2S$ and $4S$ model when $\Gamma \rightarrow \Gamma_c$. The increase of Γ leads n to remain at the half-filling even when μ is increased from zero. Therefore, the increase of Γ acts to avoid the double occupation. In the opposite limit, when $\Gamma \ll \Gamma_c$, the increase of chemical pressure tends to remove n from the half-filling which leads the $4S$ model to become closer to the classical GS model.

Therefore, it is possible to identify two regimes. In the first one, which is common to both $2S$ and $4S$ models, the region with $(+)$ is increasingly small when $\Gamma \rightarrow \Gamma_c$. In the second regime, when Γ is not close to Γ_c , there is a mixing of effects due to the presence of μ and Γ . In that case, the increase of μ produces the increase of regions where the eigenvalues λ_{\pm} are complex or negative. It should be remarked that the regions with $R < 0$ and $(-)$ are exactly those ones which occupy most part of the SG phase diagram, particularly, close to the first order boundary phase. Therefore, even as suggested in Refs. [29, 30], that the condition $R > 0$ would be enough to guarantee the SG stability, the dominance of regions with $R < 0$ and $(-)$ for certain values of Γ and μ indicate clearly an additional instability to the SG solution which must be taken into account simultaneously with that one related with the replicon.

Surely, the reliability of some results are quite limited by the use of SA , particularly, those ones close to the QCP . Nevertheless, in the first regime, the results show clearly that the tendency to decrease the region with $(+)$, when Γ is increased, can be observed even at higher temperature. This tendency would indicate that close to the QCP the analysis of λ_{\pm} could also play a more important role. However, there is no doubt that it is needed to go beyond the SA to obtain a more conclusive result close to the QCP . On the other hand, for the second regime, results are more conclusive. The effects of Γ combined with the increase of μ give an additional instability at higher temperatures. This new instability is important, particularly, close to the first order boundary where the use of SA is not so limited as in the case of a continuous transition [19].

To conclude, in this work, it has been studied in detail the behavior of λ_{\pm} in the FISG model as a function of Γ . It has been presented several evidences indicating that these particular eigenvalues of the Hessian matrix can be also source of problem to the local stability of the RS solution when Γ increases. It is clear that this results are very much restricted to the FISG model. However, one can speculate whether or not others different formulations of infinite-range Ising spin-glass model in a transverse magnetic field could experiment similar additional instabilities likewise those ones found in the $2S$ model.

One last comment must be done. Although there are strong similarities

between the $2S$ and $4S$ models in the first regime, it is not obvious that this similarity can be explained by the behavior of the n which remains at the half-filling when Γ increases. One alternative, would consist of adding to the model given in Eq. (1) a local on site repulsive Coulomb interaction with strength U . Therefore, in the limit $U \rightarrow \infty$, it would be possible to avoid locally the double occupation [13]. This approach is currently under investigation.

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